

1. NO CALCULATORS OR NOTES ALLOWED
2. SHOW PROPER CALCULUS-LEVEL WORK
3. SIMPLIFY ALL ANSWERS

Find  $\frac{d}{dx} \frac{\tanh^{-1}(x^3)}{x^2}$ . Simplify your final answer as a single fraction. SCORE: 3 / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$\begin{aligned} \frac{d}{dx} \frac{\tanh^{-1}(x^3)}{x^2} &= \frac{(\tanh^{-1}(x^3))' \cdot x^2 - \tanh^{-1}(x^3) \cdot (x^2)'}{(x^2)^2} \\ &= \frac{\left(\frac{1}{1-(x^3)^2} \cdot 3x^2 \cdot x^2\right) - (\tanh^{-1}(x^3) \cdot 2x)}{x^4} \\ &= \frac{\frac{3x^4}{1-x^6} - 2x \tanh^{-1}(x^3)}{x^4} \\ &= \frac{3x^4}{1-x^6} \cdot \frac{1}{x^4} - \left(\frac{2x}{x^4} \cdot \ln\left(\frac{1+x^6}{1-x^6}\right)\right) \\ &= \frac{3}{1-x^6} - \left[\frac{1}{x^3} \cdot \ln\left(\frac{1+x^6}{1-x^6}\right)\right] \end{aligned}$$

If  $\tanh x = -\frac{2}{3}$ , find  $\sinh x$ .

SCORE: 0 / 4 PTS

$$\tanh x = -\frac{2}{3}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = -\frac{2}{3}$$

$$\sinh x = \pm 2$$

Find  $\lim_{x \rightarrow -\infty} \tanh x$  algebraically.

SCORE: 4 / 4 PTS

$$\begin{aligned} \lim_{x \rightarrow -\infty} \tanh x &= \lim_{x \rightarrow -\infty} \frac{\sinh x}{\cosh x} = \lim_{x \rightarrow -\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{e^x}{e^x} \\ &= \lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{2x} + 1} \cdot \frac{1}{e^x} \\ &= \frac{e^{-\infty} - 1}{e^{-\infty} + 1} = \frac{\frac{1}{e} - 1}{\frac{1}{e} + 1} \\ &= \frac{\frac{1 - e}{1 + e}}{\frac{1 + e}{1 + e}} = \frac{1 - e}{1 + e} = -1 \end{aligned}$$



For this question, you may use the formulae for  $\frac{d}{dx} \sinh x$  and/or  $\frac{d}{dx} \cosh x$  without proving them.

SCORE: 2 / 7 PTS

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] Without using the logarithmic formula for  $\sinh^{-1} x$ , prove the formula for  $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$ .

$$\sinh^{-1} x = y$$

$$\sinh y = x$$

$$\frac{e^y - e^{-y}}{2} = x$$

$$e^y (e^y - e^{-y}) = (2x) e^y$$

$$e^{2y} - 1 = 2x e^y$$

$$e^{2y} - 1 - 2x e^y = 0$$

$$1 e^{2y} - 2x e^y - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1 \quad b = -2x \quad c = -1$$

$$e^y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(1)(-1)}}{2(1)}$$

$$e^y = \frac{2x \pm \sqrt{4x^2 - (-4)}}{2}$$

$$e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$e^y = x \pm \sqrt{x^2 + 1}$$

$$\sinh^{-1} x = y = \ln(x + \sqrt{x^2 + 1})$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot (x + \sqrt{x^2 + 1})'$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot (1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x)$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot (1 + \frac{x}{\sqrt{x^2 + 1}})$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \frac{x + \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}}$$

$$y' = \frac{1}{\sqrt{x^2 + 1}}$$

[b] Without using the exponential formula for  $\operatorname{sech} x$ , prove the formula for  $\frac{d}{dx} \operatorname{sech} x$ .

$$\frac{d}{dx} \operatorname{sech} x = \frac{d}{dx} \frac{1}{\cosh x}$$

$$= \frac{(1)' \cosh x - (1)(\cosh x)'}{(\cosh x)^2}$$

$$= \frac{0 - \sinh x}{\cosh^2 x}$$

$$= \frac{-\sinh x}{\cosh^2 x} \quad (1)$$

$$= \frac{-\sinh x}{\cosh x} \cdot \frac{1}{\cosh x}$$

$$= -\tanh x \operatorname{sech} x \quad (1)$$

Prove the logarithmic formula for  $\tanh^{-1} x$  given in your textbook.

NOTE: This is NOT a question about derivatives.

SCORE: 0 / 5 PTS

$$\tanh^{-1} x =$$

$$\tanh^{-1} x = \frac{1}{2} \ln$$



[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS:

**[1]**

E

**[2]**

B

**[3]**

E

**[4]**

D

**[5]**

C

**[6]**

D

**[1]** Which statement below regarding tests (quizzes, midterms, final exam) is false ?

- [a] If you continue writing on your test after the stated ending time, you will receive a 0 for that test.
- [b] There are no make-ups for missed quizzes.
- [c] The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.
- [d] If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.
- [e] If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.

**[2]** Which statement below regarding attendance is false ?

- [a] Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.
- [b] Arriving late on a quiz or midterm day will not be counted as late.
- [c] Unexcused early departures are considered absences.
- [d] If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
- [e] Attendance policies will not apply to you if you score more than 80% on every midterm.

**[3]** How much of your learning does the instructor believe comes from your daily reading and homework combined ?

- [a] 40%
- [b] 50%
- [c] 60%
- [d] 70%
- [e] 80%

**[4]** Proper use of the textbook for this class includes

- [a] understanding all the terminology used in the book
- [b] working out the given examples yourself and checking that you are able to get the same results as the book
- [c] reading the sections of the textbook before the corresponding lecture
- [d] all of the previous answers [a], [b] and [c]
- [e] some, but not all, of the previous answers [a], [b] and [c]

**[5]** If you score 120 points on Midterm 1, 140 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?  
(HINT: You are encouraged to start studying regularly early in the quarter.)

- [a] Midterm 1's score will be changed to 145 (the highest midterm score)
- [b] Midterm 1's score will be changed to  $(120 + 140 + 145) \div 3 = 135$  (the average of all midterm scores)
- [c] Midterm 1's score will be changed to  $(120 + 140) \div 2 = 130$  (the average of Midterm 1's and Midterm 2's scores)
- [d] Midterm 1's score will be changed to  $(120 + 145) \div 2 = 132.5$  (the average of Midterm 1's and the highest midterm's scores)
- [e] Midterm 1's score will be changed to  $(120 + 145) \div 2 = 132.5$  and Midterm 2's score will be changed to  $(140 + 145) \div 2 = 142.5$  (the average of each midterm's and the highest midterm's score)
- [f] no midterm scores will be changed

**[6]** If you score 140 points on Midterm 1, 120 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

- [a] Midterm 2's score will be changed to 145 (the highest midterm score)
- [b] Midterm 2's score will be changed to  $(120 + 140 + 145) \div 3 = 135$  (the average of all midterm scores)
- [c] Midterm 2's score will be changed to  $(120 + 140) \div 2 = 132.5$  (the average of Midterm 2's and Midterm 1's scores)
- [d] Midterm 2's score will be changed to  $(120 + 145) \div 2 = 132.5$  (the average of Midterm 2's and the highest midterm's scores)
- [e] Midterm 1's score will be changed to  $(140 + 145) \div 2 = 142.5$  and Midterm 2's score will be changed to  $(120 + 145) \div 2 = 132.5$  (the average of each midterm's and the highest midterm's score)
- [f] no midterm scores will be changed